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Radiative Heat Transfer Effects on a Parabolic Flow Past an Infinite Isothermal Vertical Plate in the Presence of Chemical Reaction

R.Muthucumaraswamy[*1], P.Sivakumar[2]
[*1] Department of Applied Mathematics, Sri Venkateswara College of Engineering
Irungattukottai Sriperumbudur-602105, India
[2]Department of Mathematics, P.B.College of Engineering, Irungattukottai, Sriperumbudur, Chennai 602105, India
msamy@svce.ac.in

Abstract

An exact solution of unsteady flow past a parabolic flow past an infinite isothermal vertical plate in the presence of homogeneous chemical reaction of first order has been studied. The plate temperature as well as concentration level near the plate are raised uniformly. The dimensionless governing equations are solved using Laplace transform technique. The effect of velocity profiles are studied for different physical parameters like chemical reaction parameter, thermal Grashof number, mass Grashof number, Schmidt number, Radiation and time. It is observed that the velocity increases with increasing values the thermal Grashof number or mass Grashof number. The trend is just reversed with respect to the chemical reaction parameter.

Keywords: Radiation, parabolic, homogeneous, chemical reaction, first order, isothermal, vertical plate.

Introduction

Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion device for aircraft, combustion and furnace design, materials processing, energy utilization, temperature measurements, remote sensing for astronomy and space exploration, food processing and cryogenic engineering, as well as numerous agricultural, health and military applications. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the combined effect of thermal radiation and mass diffusion

Chemical reactions can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. In well mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in the solution. In most cases of chemical, reactions, the reaction rate depends on the concentration of the species. A reaction is said to be of first order, if the rate of reaction is directly proportional to the concentration itself. Chambre and Young (1958) analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Das *et al.* (1994) studied the effect of a homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on a moving isothermal vertical plate in the presence of chemical reaction were studied by Das *et al.* (1998). The dimensionless governing equations were solved by the usual Laplace transform technique.

Natural convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method by Gupta *et al* (1979).

Kafousias and Raptis (1981) extended this problem to include mass transfer effects subjected to variable suction or injection. Soundalgekar (1982) studied the mass transfer effects on flow past a uniformly accelerated vertical plate. Mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh (1983). Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar (1984). The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo (1986). Mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion was studied by Basant Kumar Jha *et al* (1991). Agrawal et al (1998) studied free convection due to thermal and mass diffusion in laminar flow of an accelerated infinite vertical plate in the presence of magnetic filed. Agrawal et al (1999) further extended the problem of unsteady free convective flow and mass diffusion of an electrically conducting elasto-viscous fluid past a parabolic starting motion of the infinite vertical plate with transverse magnetic plate. The governing equations are tackled using Laplace transform technique.

It is proposed to study the effects of on flow past an infinite isothermal vertical plate subjected to parabolic motion in the presence of chemical reaction of first order. The dimensionless governing equations are solved using the Laplace transform technique. The solutions are in terms of exponential and complementary error function.

Mathematical Analysis

The unsteady flow of a viscous incompressible fluid past an infinite isothermal vertical plate with uniform diffusion, in the presence of chemical reaction of first order has been considered. The x'-axis is taken along the plate in the vertically upward direction and the y –axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature T_∞ and concentration C'_∞ .At time $t' > 0$, the plate is started with a velocity $u = u_0 . t'^2$ in its own plane against gravitational field and the temperature from the plate is raised to T_w and the concentration level near the plate are also raised to C'_w. A chemically reactive species which transforms according to a simple reaction involving the conecntration is emitted from the plate and diffuses into the fluid. The reaction is assumed to take place entirely in the stream. Then under usual Boussinesq's approximation for unsteady parabolic starting motion is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \quad (1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} - k_l (C' - C'_\infty) \quad (3)$$

With the following initial and boundary conditions:

$$\begin{aligned} &u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad \text{for all } y, t' \leq 0 \\ &t' > 0 : u = u_0 . t'^2, \quad T = T_w, \quad C' = C'_w \quad \text{at } y = 0 \\ &u \to 0 \quad T \to T_\infty, \quad C' \to C'_\infty \quad \text{as } y \to \infty \end{aligned} \quad (4)$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^* \sigma (T_\infty^4 - T^4) \quad (5)$$

It is assumed that the temperature differences with in the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4$$

By using equations (5) and (6), equation (2) reduces to

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} + 16a^* \sigma T_\infty^3 (T_\infty - T)$$

On introducing the following non-dimensional quantities:

$$U = u\left(\frac{u_0}{\nu^2}\right)^{1/3}, \ t = \left(\frac{u_0{}^2}{\nu}\right)^{1/3} t', \ Y = y\left(\frac{u_0}{\nu^2}\right)^{1/3},$$

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$$\theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\ Gr = \frac{g\beta(T_w - T_\infty)}{(\nu.u_0)^{1/3}},$$

$$Gc = \frac{g\beta(C'_w - C'_\infty)}{(\nu.u_0)^{1/3}}, \qquad (8)$$

$$R = \frac{16a^*\sigma.T_\infty{}^3}{k}\left(\frac{\nu^2}{u_0}\right)^{\frac{2}{3}},\ \Pr = \frac{\mu C_p}{k},$$

$$Sc = \frac{\nu}{D}$$

In equations (1), (3) and (7), reduces to

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} \qquad (9)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{\Pr}\theta \qquad (10)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - KC \qquad (11)$$

The corresponding initial and boundary conditions in dimensionless form are as follows:

$$U = 0,\quad \theta = 0,\quad C = 0 \quad \text{for all } Y, t \le 0$$

$$t > 0:\quad U = t^2,\quad \theta = 1,\quad C = 1 \quad \text{at } Y = 0$$

$$U \to 0,\quad \theta \to 0,\quad C \to 0 \quad \text{as } Y \to \infty$$

(12)

The dimensionless governing equations (9) to (11) and the corresponding initial and boundary conditions (12) are tackled using Laplace transform technique.

$$\theta = \frac{1}{2}\left[\exp\left(2\eta\sqrt{\Pr at}\right)erfc\left(\eta\sqrt{\Pr} + \sqrt{at}\right) + \exp\left(-2\eta\sqrt{\Pr at}\right)erfc\left(\eta\sqrt{\Pr} - \sqrt{at}\right)\right] \qquad (13)$$

$$c = \frac{1}{2}\left[\exp\left(2\eta\sqrt{Sc\,K\,t}\right)erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right) + \exp\left(-2\eta\sqrt{Sc\,K\,t}\right)erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right)\right] \qquad (14)$$

$$U = 2\left(\frac{t^2}{6}\left[\left(3 + 12\eta^2 + 4\eta^4\right)erfc(\eta) - \frac{\eta}{\sqrt{\pi}}\left(10 + 4\eta^2\right)\exp(-\eta^2)\right]\right)$$

$$+d\left(erfc(\eta)\right.$$
$$-\frac{\exp(bt)}{2}\left[\exp\left(2\eta\sqrt{bt}\right)erfc\left(\eta + \sqrt{bt}\right) + \exp\left(-2\eta\sqrt{bt}\right)erfc\left(\eta - \sqrt{bt}\right)\right]$$
$$-\frac{1}{2}\left[\exp\left(2\eta\sqrt{\Pr at}\right)erfc\left(\eta\sqrt{\Pr} + \sqrt{at}\right) + \exp\left(-2\eta\sqrt{\Pr at}\right)erfc\left(\eta\sqrt{\Pr} - \sqrt{at}\right)\right]$$
$$\left.+\frac{\exp(bt)}{2}\left[\exp\left(2\eta\sqrt{\Pr(a+b)t}\right)erfc\left(\eta\sqrt{\Pr} + \sqrt{(a+b)t}\right) + \exp\left(-2\eta\sqrt{\Pr(a+b)t}\right)erfc\left(\eta\sqrt{\Pr} - \sqrt{(a+b)t}\right)\right]\right)$$

$$+e\left(erfc(\eta)\right.$$
$$-\frac{\exp(ct)}{2}\left[\exp\left(2\eta\sqrt{ct}\right)erfc\left(\eta + \sqrt{ct}\right) + \exp\left(-2\eta\sqrt{ct}\right)erfc\left(\eta - \sqrt{ct}\right)\right]$$
$$+\frac{\exp(ct)}{2}\left[\exp\left(2\eta\sqrt{Sc(K+c)t}\right)erfc\left(\eta\sqrt{Sc} + \sqrt{(K+c)t}\right) + \exp\left(-2\eta\sqrt{Sc(K+c)t}\right)erfc\left(\eta\sqrt{Sc} - \sqrt{(K+c)t}\right)\right]$$
$$\left.-\frac{1}{2}\left[\exp\left(2\eta\sqrt{Sc\,K\,t}\right)erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right) + \exp\left(-2\eta\sqrt{Sc\,K\,t}\right)erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right)\right]\right)$$

Where, $a = \frac{R}{Pr}$, b= $\frac{R}{1-Pr}$, c= $\frac{KSc}{(1-Sc)}$ $d = \frac{Gr}{b(1-\Pr)}$, $e = \frac{Sc}{c(1-Sc)}$ and $\eta = \frac{y}{2\sqrt{t}}$

Results and Discussion

For physical understanding of the problem numerical computations are carried out for different physical parameters *Gr*, *Gc*, *Sc*, R,K, and *t* up on the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor. Also, the values of Prandtl number Pr are chosen such that they represent air (Pr = 0.71) . The numerical values of the velocity are computed for different physical parameters like chemical reaction parameter, Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number, Radiation, Chemical Parameters and time.

Figure 1 llustrates the effect of the concentration profiles for different values of the chemical reaction parameter ($K = 0.2, 2, 5, 10$) at $t = 0.4$. The effect of chemical reaction parameter is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the concentration increases with decreasing chemical reaction parameter.

The velocity profiles for different values of the chemical reaction parameter $(K = 0.2, 2, 5), Gr = 5 = Gc, \Pr = 0.71$ and $t = 0.2$ are shown in figure 2. It is observed that the velocity increases with decreasing chemical reaction parameter. Figure 3 demonstrates the effects of different thermal Grashof number (Gr = 2, 5), mass Grashof number (Gc = 5, 10), K=2 and Pr=0.71 on the velocity at t = 0.2. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

The velocity profiles for different values of the time $(t = 0.2, 0.4, 0.6, 0.8), K = 2, Gr = 5$ and $Gc = 5$ are presented in figure 4. The trend shows that the velocity increases with incrasing values of the time t. The effect of velocity profiles for different values of the Schmidt number $(Sc = 0.16, 0.3, 0.6), Gr = 5 = Gc, \Pr = 0.71$ and $t = 0.2$ are shown in figure 5. It is observed that the velocity increases with decrasing values of the Schmidt number.

The temperature profiles for different values of thermal radiation parameter (R= 2, 5,10), in the presence of air at time are shown in t=0.2 are presented in figure 6. The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter.

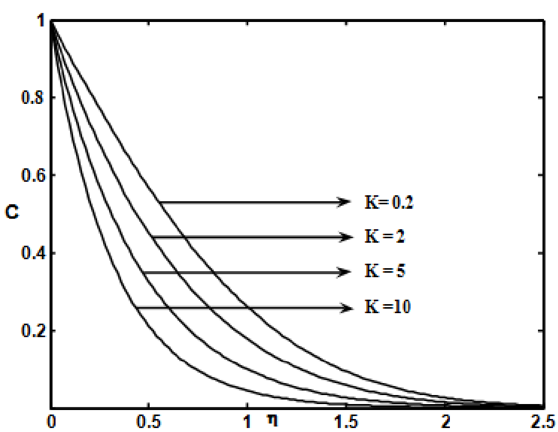

Figure 1: Concentration profiles for different values of K

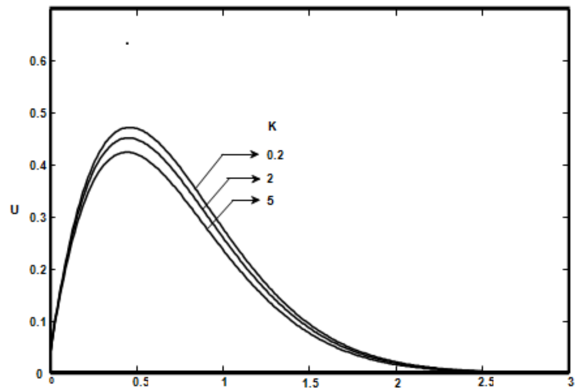

Figure 2. Velocity profiles for different K

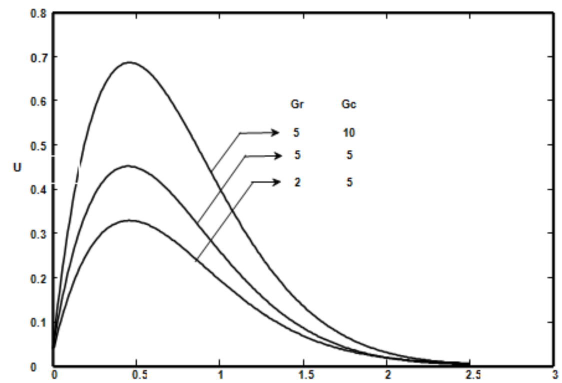

Figure 3. Velocity profiles for different Gr & Gc

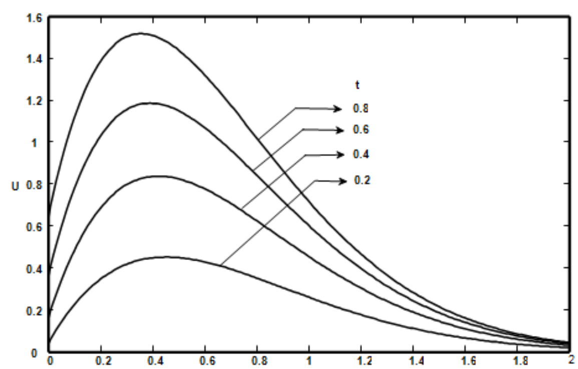

Figure 4. Velocity profiles for different t

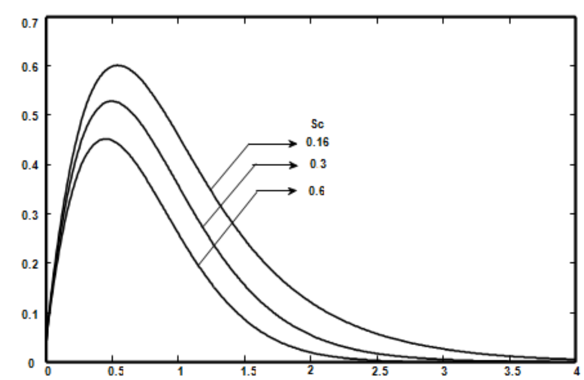

Figure 5. Velocity profiles for different Sc

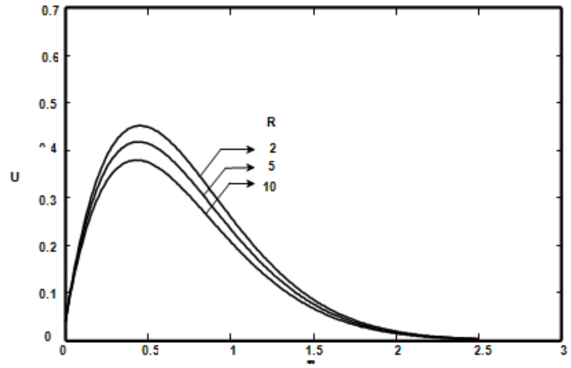

Figure 6. Velocity profiles for different R

Conclusion

An exact solution of unsteady flow past a parabolic flow past an infinite isothermal vertical plate in the presence of homogeneous chemical reaction of first order has been studied. The dimensionless governing equations are solved by the usual Laplace transform technique. The effect of the temperature, the concentration and the velocity fields for different physical parameters like chemical reaction parameter, thermal Grashof number and mass Grashof number are studied graphically. The conclusions of the study are as follows:

- (i) The velocity increases with increasing thermal Grashof number or mass Grashof number, but the trend is just reversed with respect to the chemical reaction parameter.
- (ii) The temperature of the plate increases with decreasing values of the Prandtl number.
- (iii) The plate concentration increases with decreasing values of the chemical reaction parameter.

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Nomenclature

A Constants

C' species concentration in the fluid $kg\,m^{-3}$

C dimensionless concentration

C_p specific heat at constant pressure $J.kg^{-1}.k$

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D mass diffusion coefficient $m^2.s^{-1}$
Gc mass Grashof number
Gr thermal Grashof number
g acceleration due to gravity $m.s^{-2}$
k thermal conductivity $W.m^{-1}.K^{-1}$
Pr Prandtl number
Sc Schmidt number
T temperature of the fluid near the plate K
t' time s
u velocity of the fluid in the x'-direction $m.s^{-1}$
u_0 velocity of the plate $m.s^{-1}$
u dimensionless velocity
y coordinate axis normal to the plate m
Y dimensionless coordinate axis normal to the plate

Greek symbols

β volumetric coefficient of thermal expansion K^{-1}
β^* volumetric coefficient of expansion with concentration K^{-1}
μ coefficient of viscosity $Ra.s$
ν kinematic viscosity $m^2.s^{-1}$
ρ density of the fluid $kg.m^{-3}$
τ dimensionless skin-friction $kg.m^{-1}.s^2$
θ dimensionless temperature
η similarity parameter
$erfc$ complementary error function

Subscripts

w conditions at the wall
∞ free stream conditions